A multi-period capacitated school location problem with modular equipments and age restrictions

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October 14 2011
1. **Rapidly expanding urban areas are dynamic environments**
   - Need to expand existing network of public facilities to meet anticipated increase or decrease in demand
     - schools, libraries, emergency services

2. **Closing existing facilities in areas characterized by population decline**
   - Decision making process $\rightarrow$ political decisions (bad press)
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Waddell students protest decision to close school

by ANN DOSS HELMS & MARK PRICE/Charlotte Observer

CHARLOTTE, N.C. -- Hundreds of students at E.E. Waddell High School held hands and stood outside the school Wednesday morning, refusing to go inside as a silent protest to Charlotte-Mecklenburg Schools’ decision to close the school.

In a Tuesday night meeting marked by split votes, angry protests and accusations of racism, the Charlotte-Mecklenburg school board approved a sweeping plan to close 10 schools and make other dramatic changes.
**Motivation**

1. **Location models** → tools for regional and urban planners and decision makers
   - Focus is on multi-period school locations
   - Explicit temporal component
   - Each student must be assigned to a school
   - Budget determines number of schools ($p$-median)
   - Limited capacities

2. **Dynamic problems** for different time periods
   - Ability to handle school closure and addition to existing network

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Existing objectives in the literature

- General objectives in public facility location models:
  1. Minimize total travel distance to the facility $j$ (Hakimi 1964)
  2. Minimizing necessary infrastructure while keeping a certain level of coverage (Toregas et al. 1971)
  3. Each individual $i$ is assigned to its closest facility $j$, or
  4. Facility $j$ is located within a suitable distance $d_{max}^i$ from $i$
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**Existing objectives in the literature**

1. **Dynamic models** have received a lot of attention:
   - Drezner (1995b) suggests a progressive $p$-median (no relocation of facilities).

2. Significant body of literature in the school location
     - Handling opening of new facilities, expansion, reduction.
     - Age of facility, number of time periods.
     - Uncertainty during time periods.
     - Distance penalty beyond threshold.
     - Different growth scenarios.
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Research Objectives

• **Dynamic facility** problem should address changes in population counts.

  1. number of open facilities $p$ is increased or decreased at each time period or is controlled by cost function.

  2. Each facility has a pair of **minimum** and **maximum** capacity constraints.

     • Maximal capacity can be increased with addition of **modular** equipments.

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Formulation

Notation: indices, sets and parameters

Indices and sets:

- \( i, I \): index and set of demand areas (aggregated population)
- \( j, J \): index and set of candidate locations (school)
- \( m, M \): index and set of time period (or window)
- \( J^N \): set of new candidate locations, \( J^N \subset J \), and \( J^E \) the set of existing facilities, \( J^E \subset J \)

Travel costs:

- \( t_{ijm} \): travel time between an individual \( i \) and facility \( j \) at time period \( m \).
- \( t_{im}^{\text{max}} \): maximum travel time an individual is willing to travel
- \( t_{im}^{\text{max}} > \min_{j \in J} t_{ijm} \).
- \( c_{ijm} \): cost to travel from \( i \) to \( j \) at time \( m \);

\[
c_{ijm} = \begin{cases} 
\alpha t_{ijm} & \text{if } t_{ijm} \leq t_{im}^{\text{max}} \\
\alpha t_{im}^{\text{max}} + (t_{ijm} - t_{im}^{\text{max}})^\beta & \text{if } t_{ijm} > t_{im}^{\text{max}} 
\end{cases} 
\]

Facility costs:

- \( c^{++} \): leasing cost of a (mobile) class unit/trailer
- \( c_{fjm} \): cost of operating a school \( j \) at time \( m \)
- \( c_{sm} \): marginal cost per student at time \( m \).
- \( c_{Njm} \): cost to open a new school \( j \) at time \( m \): set-up or construction cost.
- \( c_{Ejm} \): cost to close an existing school \( j \) at time \( m \).

Capacities and additional parameters:

- \( z_{jm}^+ \): max. capacity for facility \( j \) at time \( m \).
- \( K_{jm} \): max. number of additional units at \( j \) at time \( m \).
- \( z^{++} \): capacity of a soft unit (e.g. 30 students).
- \( a_{im} \): demand at location \( i \) at time \( m \)
- \( e_{jm} \): age of existing facility \( j \) at time \( m \)
- \( E \): minimal age for a facility to close
- \( w_m \): weight reflecting importance of time period.
- \( \gamma_{1,2} \): importance given to travel and school operating costs, respectively.
Maximal traveled distance $d_{im}^{max}$ or $t_{im}^{max}$

$t_{ijm}$: travel time between an individual $i$ and facility $j$ at time period $m$.

$t_{im}^{max}$: maximum travel time an individual is willing to travel $t_{im}^{max} > \min_{j \in J} t_{ijm}$.

$N_{im} = \{ j \in J | t_{ijm} \leq t_{im}^{max} \}$: Set of facilities within acceptable travel time ($i$).

$c_{ijm}$: cost to travel from $i$ to $j$ at time $m$; $c_{ijm} = \begin{cases} \alpha t_{ijm} & \text{if } t_{ijm} \leq t_{im}^{max} \\ \alpha t_{im}^{max} + (t_{ijm} - t_{im}^{max}) \beta & \text{if } t_{ijm} > t_{im}^{max} \end{cases}$

An individual $i$ at time $m = 1$ can choose among different facilities. Once assigned to a facility beyond a travel distance $d_{i}^{max}$, the impedance increases exponentially with rate $\beta$. 
Notation: decision variables

\[ X_{ijm} = \begin{cases} 
1 & \text{if demand } i \text{ is assigned to } j \text{ at time } m \\
0 & \text{otherwise} 
\end{cases} \]

\[ Y_{jm} = \begin{cases} 
1 & \text{if location } j \text{ is opened at time } m \\
0 & \text{otherwise} 
\end{cases} \]

\[ B_{jm} = \text{modular units necessary at } j \text{ at time } m \]
Formulation: objective function

Minimize \( F = (\gamma_1 \cdot F_1) + (\gamma_2 \cdot F_2) \)

\[ F_1 = \left[ \sum_{i \in I} \sum_{m \in M} \left( \sum_{j \in J} w_m \cdot c_{ijm} \cdot a_{im} \cdot X_{ijm} \right) \right] \]

- transportation costs

\[ F_2 = \left[ \left( \sum_{j \in J} \sum_{m \in M} c_{fjm} \cdot Y_{jm} \right) + \left( \sum_{j \in J^N} \sum_{m \in M \setminus \{1\}} c_{Nm} \cdot (Y_{jm} - Y_{j,m-1}) \right) + \right. \]

- fixed costs

- opening cost

\[ \left( \sum_{j \in J^E} \sum_{m \in M \setminus \{1\}} c_{Ejm} \cdot (Y_{j,m-1} - Y_{jm}) \right) + \left( \sum_{i \in I} \sum_{j \in J} \sum_{m \in M} c_{sm} \cdot a_{im} \cdot X_{ijm} \right) + \]

- closing cost

- student cost

\[ \left( \sum_{j \in J} \sum_{m \in M} c_{m}^{++} \cdot B_{jm} \right) \]

- leasing cost
Assignment, capacity, age, closing constraints

\[ \sum_{j \in J} X_{ijm} = 1 \quad \forall i \in I, \forall m \in M \]

\[ X_{ijm} \leq Y_{jm} \quad \forall i \in I, \forall m \in M, \forall j \in J \]

\[ \sum_{i \in I} a_{im} \cdot X_{ijm} \leq (e_{jm}^+ \cdot Y_{jm}) + (N_{jm} \cdot z^{++}) \quad \forall j \in J, \forall m \in M \setminus \{1\} \]

\[ B_{jm} \leq A \cdot Y_{jm} \quad \forall j \in J, \forall m \in M \setminus \{1\} \]

\[ B_{jm} \leq K_{jm} \quad \forall j \in J, \forall m \in M \setminus \{1\} \]

\[ \bar{E} - e_{jm}^E \leq A \cdot Y_{jm} \quad \forall j \in J^E, \forall m \in M \setminus \{1\} \]

\[ Y_{j,m-1} \leq Y_{jm} \quad \forall j \in J^N, \forall m \in M \setminus \{1\} \]

\[ Y_{jm} \leq Y_{j,m-1} \quad \forall j \in J^E, \forall m \in M \setminus \{1\} \]

\[ X_{ijm}, Y_{jm} \in 0, 1 \quad \forall i \in I, \forall j \in J, \forall m \in M \]

\[ B_{jm} \in Z^+ \quad \forall j \in J, \forall m \in M \setminus \{1\} \]
GIS

Linking GIS to solvers

Generates locations of:
- demand nodes
- facilities
- city centers

Computes:
- distances

Visualization:
- capacities
- assignments (spiders)

ArcGIS

python IDLE

File exchange:
LP, results

calls reads

Lingo

The semi-loosed coupled system. A python script is built in the IDLE environment, automatically calling Lingo. The exchange of information is carried out through text files.

Growth Scenarios

- We consider four time periods $t_1$, $t_2$, $t_3$ and $t_4$
  1. In base year $t_1$, we simulate population $\sim U(5, 100)$
  2. Population growth is not uniform
     - $\sum_{i,m=1} a_{im} = 3780$,
     - $\sum_{i,m=2} a_{im} = 5670$,
     - $\sum_{i,m=3} a_{im} = 8503$,
     - $\sum_{i,m=4} a_{im} = 12758$. 
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\]
Calibration of cost parameters

\( c_{jm} \): $710,000.
\( c_{sm} \): $7,200.
\( c_{Njm} \): $3,000,000 for a school of 60 acres.
\( c^{++} \): $10,000 per year (leasing), which includes rent, heating, maintenance.
\( z^{++} \): 25 students per unit.

\( c_{Ejm} \): $1: this cost can be negative, that is there is a benefit to close a school. In this paper, we take a symbolic cost of one unit.
\( K_{jm} \): upper-limit of 100 units.
\( \bar{E} \): 32 years.
Growth Scenarios

<table>
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<tr>
<th>time 1</th>
<th>time 2</th>
<th>time 3</th>
<th>time 4</th>
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<td>monocentric direct</td>
<td>monocentric inverse</td>
<td>monocentric inverse</td>
<td>uniform</td>
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</tbody>
</table>

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Growth Scenarios

- **Polycentric Direct**
  - Time 1
  - Time 2
  - Time 3
  - Time 4

- **Polycentric Inverse**
  - Time 1
  - Time 2
  - Time 3
  - Time 4

- **Demand Node**
- **City Center**

- **Proximity Buffer**
  - 10, 20, 30 km

- **Demand**
  - 52, 90, 130, 200

- **Scale**
  - 0, 25, 50 km

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Dynamic School Location

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Time period weights

\[ F_1 = \left[ \sum_{i \in I} \sum_{m \in M} \left( \sum_{j \in J} w_m \cdot c_{ijm} \cdot a_{im} \cdot X_{ijm} \right) \right] \]

\[ w_m = \frac{f}{(1 + g)^m}; \quad \sum_{m \in M} w_m = 1 \tag{1} \]

<table>
<thead>
<tr>
<th></th>
<th>case a</th>
<th>case b</th>
<th>case c</th>
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<td>2</td>
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<tr>
<td>g</td>
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<td>( w_4 )</td>
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<td>.02555034</td>
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</table>
Initial assignment

(a) location, ID and age of facilities

(b) capacitated p-median solution (t1)

- demand
- facilities: open
- closed

allocation: 1

capacity usage: 1

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Solution from different growth scenarios

(time 1) (time 2) (time 3) (time 4)

direct monocentric

inverse monocentric

direct polycentric

inverse polycentric
Impact of $\beta$: putting extra-pressure beyond $t_{im}^{max}$
Impact of weights

\[ \text{time 1} \hspace{2cm} \text{time 2} \hspace{2cm} \text{time 3} \hspace{2cm} \text{time 4} \]

- \( \gamma_1 = 0.5, \beta = 2.5 \)
- \( \gamma_1 = 0.9, \beta = 0 \)
Minimal age for school closure

- No age limit
- 32 years age limit
- 60 years age limit

Time 1, Time 2, Time 3, Time 4
This paper presents a multi-period location problem

• Capacity, **age** constraints and **modular** equipment.
• Model is flexible (time-varying weights for uncertainty)

Generate small instances for different growth scenarios

• Heuristics are necessary for larger problems

Other modeling concerns . . .

• Quantifying transportation costs (environmental impact of modal choice) and travel time
• Leasing modular equipments → long-term solution?
• Splitting the demand (and disruption to the student)
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Discussion

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questions ... comments ... concerns ... suggestions
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