Splitting and lasing of whispering gallery modes in quantum dot micropillars

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Abstract: We have studied the whispering gallery mode (WGM) resonances of GaAs/AlGaAs microcavity pillars containing InAs quantum dots. High quality factor WGMs are observed from a wide range of pillars with diameters from 1.2 to 50 µm. Multimode lasing with sub-milliwatt thresholds and high beta-factors approaching unity is observed under optical pumping in a 4 µm diameter pillar. Mode splitting is observed in WGMs from pillars with diameters of 5 µm, 20 µm and 50 µm. We develop a model in which the mode splitting in the larger pillars is caused by resonant scattering from the quantum dots themselves. The model explains why splittings are observed in all of the larger pillars and that the splitting decreases with increasing wavelength. Numerical simulations by COMSOL confirm that the model is plausible. This mechanism of splitting should be general for all circular resonant structures containing quantum dots such as microdisks, rings, toroids, and microspheres.

References and links
1. Introduction

Whispering gallery modes (WGMs) in optical microresonators have been thoroughly investigated for applications in lasers, sensors, and cavity quantum electrodynamics (QED) [1–6]. In semiconductor physics, WGMs have been widely studied in microdisks, originally for applications in lasers [7–9]. More recently, interest has focussed on quantum dot microdisks [10, 11], with particular emphasis on lasing [12–14] and cavity QED phenomena, such as the Purcell effect [15] and strong coupling [16]. In parallel with this work, there has also been much interest in quantum dots in micropillars [18]. Here, the dots are embedded within the microcavity defined by two distributed Bragg reflector (DBR) mirrors, and small volume (V) microresonators are fabricated by etching pillars with micron-sized diameters. However, it is usually the axial (vertical) modes of the micropillars rather than WGMs that are the focus of attention.

It has recently been discovered that quantum dot (QD) micropillars can also support high Q (quality factor) WGMs in addition to their axial modes [18, 19]. These WGMs are confined in the vertical direction by the DBR mirrors and in the horizontal direction by the pillar walls. This leads to $Q/\sqrt{V}$ ratios that are comparable to those of the axial modes, making them highly suitable for the observation of cavity QED phenomena [18]. Moreover, their lasing properties have been found to be stable at high pump powers [20], and the modes have been proposed as a potential source of terahertz radiation [21]. All of these factors make their further study highly desirable.

In this paper we report a new experimental geometry that has allowed us to study the WGMs of quantum dot micropillars in far greater detail than in our previous work [18]. This has allowed us to investigate the lasing behaviour of the WGMs and to carry out a systematic study of the mode structure in a wide range of pillars. A specific question that we address is the cause of the WGM splitting that is sometimes observed [19, 20]. This mode splitting is generally attributed to scattering centres that break the symmetry and hence lead to the formation of symmetric and antisymmetric standing waves having different overlap with the light-scattering particles. We present results here that support the proposal that the quantum dots themselves can act as the scattering centres that induce the splitting [22]. This implies that WGM splitting in quantum dot micropillars is possible even if the processing is perfect. Although this mechanism of WGM splitting was largely overlooked in previous experimental studies, we believe that it is intrinsic for all resonant structures that incorporate quantum dots, such as microdisks, rings, toroids, and spheres.

The paper is organized as follows. The new experimental geometry and its advantages are described in Section 2. Section 3 outlines the characterization of the WGMs, while Section 4 reports their lasing properties. The mode splitting is investigated in detail in Section 5, and the
conclusions are given in Section 6.

2. Samples and experimental methods

The quantum dot microcavity wafer was grown by molecular beam epitaxy on a GaAs substrate. The cavity was designed to support a single axial mode of wavelength ($\lambda \sim 950$ nm). The bottom and top DBR mirrors were formed respectively from 27 and 20 periods of GaAs/Al$_{0.8}$Ga$_{0.2}$As pairs. The $\lambda$-cavity region was formed by a GaAs spacer with a thickness of 276 nm. A single layer of InAs self-assembled quantum dots of density $\sim 5 \times 10^{9}$ cm$^{-2}$ was grown at the centre of the GaAs spacer to serve as the internal light source and laser gain medium. Micropillar structures with diameters ranging from 0.8–50 $\mu$m were then fabricated by electron beam lithography (EBL) and inductively coupled plasma (ICP) etching. Further details of the fabrication process may be found in ref. [23]. A scanning electron microscope (SEM) image of a typical micropillar is given in Fig. 1.

![Fig. 1. SEM image of a typical micropillar, with a schematic showing the excitation and collection paths for the off-axis micro-photoluminescence technique. 'Horizontal' and 'vertical' polarizations are defined, respectively, by whether the electric field is perpendicular to, or has a component parallel with, the 'vertical' pillar axis.](image)

The resonant modes of the micropillars were investigated by the micro-photoluminescence ($\mu$PL) technique. All of the measurements were carried out at $\sim 4$ K in a continuous-flow liquid helium cryostat. The excitation laser was focussed to a diameter of approximately 1 $\mu$m on the top surface of the micropillars by a long-working-distance 50$\times$ microscope objective with a numerical aperture of 0.42. The resulting photoluminescence (PL) was imaged onto the slits of a spectrograph and detected with a silicon charge coupled device (CCD).

The axial modes of micropillars are routinely measured by collecting the emission from the top of the cavity with the same microscope objective as that used to focus the laser. This geometry is not suitable for the observation of WGMs, and certain modifications have to be made. In our previous work, the sample was cleaved and rotated in order to present the pillar sidewall to the laser and collection lens [18]. This allowed direct excitation of the cavity region and subsequent collection of the WGM emission, but at the risk of total sample destruction during the cleaving process. In order to avoid this risk, and hence investigate a larger number of pillars, a different approach was employed here in which the emission was collected from the side of the pillar after excitation on the top, as shown schematically in Fig. 1.

The experimental arrangement for the modified $\mu$PL geometry is shown in Fig. 2. The sample was mounted flat in the cryostat and excited on its top surface, with the emission collected by a second lens mounted at $\sim 60^\circ$ to the pillar axis. A custom-designed cryostat cover was manufactured for this purpose. The collection lens had a diameter of 8 mm and focal length...
of 20 nm. The small lens diameter was chosen to allow maximum angular deviation from the pillar axis before coming into contact with the cryostat cover. The maximum collection angle of 60° was found to be sufficient to allow collection of the WGMs whilst rejecting the axial cavity modes. Good signal strength was obtained in spite of the small numerical aperture (0.20) on account of the strong intensity of the WGMs.

Three different continuous wave (CW) laser sources were used for the experiments. Most of the measurements were made with a 633 nm HeNe laser with a power of 2 mW. However, this laser had insufficient power to excite WGMs in the 20 µm and 50 µm pillars, and so was replaced by a 650 nm diode laser with a power of 25 mW. A CW Ti:sapphire laser was also used to excite some of the samples at longer wavelengths.

3. Characterization of the whispering gallery modes

The WGMs were observed as a comb-like series of regularly-spaced peaks superimposed over the broad, inhomogeneous emission spectrum of the quantum dots, as in previous work [18]. Figure 3(a) shows a typical result for a 4 µm diameter pillar excited by a 633 nm HeNe laser, while Fig. 3(b) plots the free spectral range (FSR) against the inverse of the radius $R$ for diameters ranging from 1.2 µm to 50 µm. The linear scaling of the FSR with $R^{-1}$ is consistent with the simplest model in which an integer number of wavelengths is required to fit within the circumference. The FSR in energy units ($\Delta E$) is then given by:

$$\Delta E \approx \frac{hc}{2\pi R n_{\text{eff}}},$$

where $n_{\text{eff}}$ is the effective refractive index. Detailed fitting of the WGM wavelengths requires that the dispersion of $n_{\text{eff}}$ is included. An excellent fit was obtained with $n_{\text{eff}} = 3.693 - 1.052E + 0.610E^2$, where $E$ is the photon energy in eV units [19], as shown by the red line in Fig. 3(b).

Polarization measurements on the WGMs were carried out by making polar plots of the emission intensity, according to the polarization geometry defined in Fig. 1. In microdisks both transverse electric (TE) and transverse magnetic (TM) modes are observed [10]. However, in agreement with our previous work [18], all of the modes from our pillars were found to be strongly polarized perpendicular to the axis — i.e. “horizontally” (see Fig. 1) — which implies that they originate from TE modes. The dominance of the TE modes is caused by the orientation of the dipoles within the plane of the pillar [24, 25], which means that the quantum dots do not couple efficiently to TM modes [19].
The precise determination of the quality factor ($Q$) of the modes is complicated by the fact that the mode line width narrows as the excitation power is increased. In the literature it is common to refer to three different regimes [26]. The “cold-cavity” regime occurs at low powers, when the $Q$-factor is limited by the absorption of the gain medium [8]. In our case, this means that the $Q$ measured at low powers is affected by the absorption of the dots themselves. On increasing the power, the “empty-cavity” regime is reached when the quantum dots become transparent, and $Q$ is limited by the scattering of photons by cavity edge roughness [10]. Finally, in the “hot-cavity” regime at the highest powers, the line width decreases further, as expected for a laser operating above threshold [14]. In what follows we mainly discuss the empty-cavity quality factor ($Q_{\text{empty}}$), since this is the parameter that best specifies the intrinsic quality of the resonator. However, since it is difficult to identify the transparency point precisely, the values we quote for $Q_{\text{empty}}$ are the $Q$ values deduced from the linewidth observed around threshold.

The inset to Fig. 3(b) shows the estimated empty-cavity $Q$ values for pillars with diameters ranging from 4 to 20 $\mu$m. We observe a general trend for $Q$ to increase with increasing diameter, on account of the reduction in radiation losses and the improvement in the relative surface smoothness. The $Q$ values of both the 10 and 20 $\mu$m pillars are over 30,000, which is indicative of very low surface roughness. The 50 $\mu$m pillars showed still higher $Q$ values, with a resolution-limited line width of 0.0115 nm (equivalent to an apparent $Q$ of 86,000) measured at 989 nm. It was not possible to observe lasing for these pillars, and so it is not clear whether this value corresponds to the cold- or empty-cavity $Q$.

The $Q$ factors of the WGMs are comparable to those of the axial modes in pillars of similar size. For example, a $Q$ of around 12,000 is fairly typical for the fundamental axial mode of our 3 $\mu$m diameter pillars [27], which compares favourably to the empty-cavity $Q$ of WGMs in the 4 $\mu$m diameter pillars. These high $Q$ values make the WGMs suitable for observing low-threshold lasing, as discussed in the next section.

4. Whispering gallery mode lasing

The lasing behaviour of the WGMs was investigated by measuring the emission spectrum as a function of CW excitation power. Figure 4 shows the power dependence of the 958 nm WGM from a 4 $\mu$m diameter pillar when excited by either the HeNe laser or Ti:sapphire laser running at 800 nm. As shown in Fig. 3(a), this was the strongest WGM mode observed at low power from the pillar.
The nonlinear dependence of the emission on the pump power in Fig. 4 is suggestive of lasing, with clear saturation observed for 800 nm pumping at the highest powers. Detailed analysis of the input-output curve on a log-log scale indicates that the lasing threshold was approximately 200 µW when pumped at 633 nm. (See Fig. 5 and its discussion.) A similar analysis for 800 nm pumping (not shown) gives a lower threshold of around 70 µW. The reduction in the threshold for 800 nm pumping can be attributed to the reduced absorption of the top DBR mirror and hence higher pumping efficiency. The thresholds measured here are lower than those reported previously for quantum dot micropillar WGMs [19,20], and compare favourably with those observed from other GaAs based quantum dot/cavity systems [13, 28]. The reduced thresholds compared to refs [19] and [20] are presumably caused by the more efficient pumping through the uncoated top mirror, and also the high quality factors of our pillars.

The inset to Fig. 4 shows the narrowing of the 958 nm mode with increasing power. The increase of the $Q$ from around 5,000 in the cold-cavity regime to $\sim 10,000$ for the empty cavity is apparent. The narrowest line width observed for this mode was 75 µeV.

At the highest pump powers, simultaneous lasing was observed for the modes at 923, 940, 958 and 978 nm. This multimode lasing behaviour is a consequence of the inhomogeneous nature of the quantum dot gain medium. The gain was largest for the 958 nm mode which lies closest to the peak emission of the quantum dot ensemble. However, the threshold decreased steadily with increasing wavelength, and so was lowest for the 976 nm mode. The decrease of the threshold with wavelength can be explained by the fact that the dots in the long wavelength tail are larger and therefore couple more efficiently to the electric field of the mode [12].

Jaffrennou et al. have demonstrated that micropillar WGM lasing behaviour is stable at high powers and that a much smaller thermal red-shift is observed compared to microdisks [20]. This reduced red shift is a consequence of the improved heat-sinking properties of the bottom DBR mirror compared to the pedestal of the microdisk. In our experiments the maximum red shift under 633 nm pumping was 0.24 nm, which is comparable to the largest red shift reported in ref. [20].

The $\beta$-factor for the lasing mode can be estimated by fitting the input-output characteristic on a log-log scale [29]. Following Jaffrennou et al. [20], we model the relationship between the
pumping rate $p$ and the mean photon number $\eta$ as:

$$p = \frac{\Gamma}{\beta} \left[ 1 + 2\xi + 2\beta(\eta - \xi) \right] \frac{\eta}{1 + 2\eta},$$  \hspace{1cm} (2)

where $\Gamma = 2 \times 10^{11} \text{s}^{-1}$ is the cavity loss rate (equivalent to an empty-cavity $Q$ of 10,000) and $\xi$ is the mean photon number at transparency. According to the two-level atom model developed in ref. [30], $\xi$ is subject to the constraint $\xi \geq 0.25$, and can be related to $\beta$ through:

$$\xi = \frac{N\beta}{2\tau\Gamma},$$  \hspace{1cm} (3)

where $N$ is the total number of lasing dots, and $\tau$ is the radiative lifetime. The fitting of the data requires two further parameters to scale the pumping rate to the external pump power and the mean photon number to the emission intensity.

Excellent fits to the data in Fig. 4 for 633 nm pumping are obtained with $0.56 \leq \beta \leq 0.72$ for $\xi$ in the range 0.25–1.0. Figure 5 shows the fit with $\beta = 0.61$ and $\xi = 0.4$. The lasing threshold of 200 $\mu$W can be read from this graph as the power for which $\eta = 1$. It is noteworthy that the value of $\beta$ reported here is significantly higher than that of 0.075 in ref. [20], and also that of 0.09–0.2 quoted for WGMs in a GaAs-based microdisks [9, 31].

It should be pointed out that the experimental data is not good enough to obtain a reliable value of $\xi$ from the fitting procedure, but that this has a relatively small effect on the values of $\beta$ and the lasing threshold that are obtained. The value of $\xi = 0.4$ chosen for Fig. 5 corresponds to $N/\tau = 260 \text{ns}^{-1}$ in eqn 3. The maximum possible value of $N$ can be estimated from the modal volume $V_c \sim 0.3 \mu m^3$ [18] and the dot density $(5 \times 10^9 \text{cm}^{-2})$. If we assume that the mode approximates to a torus around the outer edge of the pillar, we then obtain $N \sim 100$, which implies $\tau \sim 0.4 \text{ns}$. The actual values of both $N$ and $\tau$ are probably both smaller, since many of the dots will not be in resonance with the lasing mode, and the value of $\tau$ will be strongly reduced by the Purcell effect [32].

The value of $\beta$ can be compared to that estimated from the Purcell factor $F_P$ according to [33]:

$$\beta = \frac{F_P}{(1 + F_P)},$$  \hspace{1cm} (4)
where:

\[ F_p = \frac{3}{4\pi^2} \frac{Q_{\text{empty}}(\lambda/n_{\text{eff}})^3}{V_c}. \]  

(5)

Here, \( Q_{\text{empty}} \approx 10,000 \) is the empty-cavity \( Q \), \( \lambda \) is the wavelength, and \( n_{\text{eff}} \) is the effective refractive index. This gives \( \beta \approx 0.98 \) at 958 nm for \( n_{\text{eff}} = 3.34 \). The lower value of \( \beta \) deduced from the data can be explained by homogeneous broadening of the emission lines under non-resonant pumping [20].

The power-dependent measurements discussed here were repeated for other pillars with diameters in the range 4–20 µm. (Pillars with diameters lower than 4 µm did not support WGM lasing.) In all cases, the line widths measured at high powers were smaller than those at threshold by a factor of approximately 2/3. This gave line widths around 30 µeV for the 10 and 20 µm pillars well above threshold. (See inset to Fig. 8 for the line widths of the 20 µm pillar.)

5. **Splitting of whispering gallery modes**

Split WGMs have been reported from a variety of microresonator systems, for example: microdisks [13], microtoroids [34], and microspheres [35]. The splitting is generally caused by the formation of a superposition of clockwise and counter-clockwise travelling waves, the degeneracy of which is lifted by scattering [36]. The scattering centres can be located either within the resonator (e.g. the quantum dots [22]) or on the outside (e.g. nanoparticles on the pillar surface [35]). In the latter case, the WGMs are coupled evanescently to the scattering centres and calibration of the splitting provides a means to measure the nanoparticle size [35]. Moreover, the splitting can be deliberately induced and its magnitude varied by controlled evanescent coupling to an external perturbation such as a wavelength-scale tapered fibre probe [34].

In this section we report the splitting of WGMs in micropillars with diameters of 5 µm, 20 µm, and 50 µm. Split micropillar WGMs were reported in refs [19] and [20], but not in our previous work [18]. We shall show here that the mode splittings in some of our pillars can be explained by assuming that it is the quantum dots themselves that act as the scattering centres.

Two key experimental observations that support this model are that splittings are observed in all of our larger pillars, and that the magnitude of the splitting roughly follows the ensemble spectrum of the dots.

### 5.1. Splitting in 5µm diameter pillars

The WGMs of some, but not all, of the 5 µm pillars were observed to split into doublets. A typical example is shown in Fig. 6(a). In this case, a splitting of 0.33 nm is observed, which should be compared to the FSR of 16 nm. This splitting is of a similar magnitude to that reported by Nowicki-Bringuier et al. for a 2.85 µm pillar [19].

In order to investigate the origin of the splitting, scanning electron microscope (SEM) images were taken of two 5 µm pillars, one of which showed splitting and the other not. Both pillars were found to be slightly elliptical, with the pillar that had the higher ellipticity showing no splitting. We thus discount ellipticity as the origin of the splitting.

The SEM of a pillar that exhibited mode splitting is shown in Fig. 6(b). A pronounced fracturing can be observed in the bottom DBR. Scattering from this surface roughness is an obvious candidate to explain the splitting, although it should be pointed out that defects were also present (but to a lesser degree) in pillars that had unsplit WGMs. The correlation with surface roughness was corroborated by comparing the \( Q \) values. It was found that the cold-cavity \( Q \) values of unsplit WGMs was round 20,000, but those of pillars that exhibited splitting were reduced by a factor of \( \sim 2 \). The SEM images thus make it plausible to argue that the doublets in the 5 µm pillars are caused by surface roughness. However, we cannot rule out that the quantum
dots themselves might be acting as the scattering centres here just as for the larger pillars. This point is discussed further in Section 5.3.

5.2. Mode Splittings in 20 μm and 50 μm diameter pillars

A WGM spectrum from a 20μm pillar showing doublets is presented in figure 7(a). The inset shows the doublet centred at 984.8 nm in more detail. The mode splitting Δλ deduced from this PL spectrum is plotted against wavelength in Fig. 7(b). It is apparent that there is a strong variation with wavelength, with Δλ decreasing by approximately a factor of two from 955 to 1005 nm. The grey curve in Fig. 7(b) is described below in the discussion of eqn 6.

In contrast to the 5μm pillars, WGM doublets were observed from all of the 20μm pillars that were studied. The mode splittings were found to be very similar in all cases. Furthermore, the absolute magnitude of the splitting, and its ratio relative to the FSR, was much larger. This all suggests that the mode splitting is intrinsic to the structure rather than caused by processing-dependent defects.
In ref. [22] it is suggested that the quantum dots themselves can act as the scattering centres that induce the mode splitting. Since the quantum dots are embedded within the cavity, they can have much larger overlaps with the mode than equivalent nanoparticles deposited on the surface. This increases the likelihood of observing splitting rather than just broadening, as was observed when a large number of CdSe/ZnS nanocrystals were deposited on the surface of a fused-silica microsphere [37]. In Section 4 it was estimated that there were approximately 100 quantum dots within the WGM of a 4 µm diameter pillar, and this number is expected to increase approximately linearly with the pillar diameter. We therefore need to consider the scattering caused by multiple, randomly-positioned quantum dots. A related scenario was considered by Chantada et al. in ref. [38]. It was shown there that the mode splitting is expected to increase in proportion to $\sqrt{N}$, which increases the chance of observing a measurable splitting in larger pillars. We therefore propose here that scattering by QDs is the cause of the observed splittings. In Section 5.3 below we present numerical simulations that indicate that this model is plausible.

Since the QDs are necessarily present in all the pillars, the model readily explains why mode splitting is observed in all of the 20 µm pillars. Furthermore, the decrease of $\Delta \lambda$ with wavelength can be explained by making the assumption that the scattering cross-section of the dots takes the following form:

$$\sigma(\lambda) = \sigma_0 + \sigma_{\text{res}}(\lambda).$$  \hspace{1cm} (6)

Here, $\sigma_0$ represents a non-resonant term that arises from the refractive index difference associated with the QDs and depends only weakly on $\lambda$, while $\sigma_{\text{res}}(\lambda)$ represents a resonant term arising from the optical transitions of the dots [39]. This resonant term is expected to have a strong wavelength dependence that is related to the absorption of the dots. If we assume that the scattering has a $\sqrt{N}$ dependence, then we might expect the resonant term to follow the square root of the quantum dot spectrum. The grey curve in Fig. 7(b) shows the square root of the PL spectrum in Fig. 7(a) with an offset to account for non-resonant scattering. It is apparent that this is a reasonable approximation to the wavelength dependence of the splitting, which supports the model that we are proposing here.

Fig. 8. Power dependence of the split modes shown in the inset to Fig. 7(a) with CW excitation at 650 nm. The inset shows the power dependence of the line widths.

The power dependence of the two split modes shown in the inset to Fig. 7(a) is presented in Fig. 8, with the inset showing the mode line widths. A nonlinear increase in the intensity
together with an accompanying line narrowing is observed for both modes, as consistent with lasing. The threshold is higher than for the 4 \( \mu \)m pillar shown in Fig. 4 on account of the larger modal volume. At high powers, the longer wavelength mode (i.e. mode B) was found to have the higher intensity and narrower line width. The higher gain and narrower line width for the longer wavelength mode is typical for most of the split WGMs observed from the 20\( \mu \)m diameter pillars. This is consistent with the fact that the index of the dots is higher than the micropillar matrix, which means that the symmetric mode has the larger overlap with the QDs [22], and hence stronger coupling to them.

Figure 9(a) presents a typical PL spectrum for a 50 \( \mu \)m pillar excited at 650 nm by the high power diode laser, with an expanded spectrum for the modes around 978 nm shown in the inset. It is apparent that each WGM is split into a quadruplet rather than a doublet. For convenience, we label these modes as 1–4 in order of increasing wavelength, as shown in the inset.

Fig. 9. (a) PL spectrum of WGMs taken from a 50\( \mu \)m diameter pillar. The inset gives a magnified section of the spectrum around 978 nm. The components of the quadruplet are numbered 1, 2, 3 and 4 with ascending wavelength. (b) Mode splitting between the four components of the quadruplet. The schematic diagram illustrates a “triangle” mode which could cause the periodic modulation of the background spectrum in (a).

The quadruplet WGM structure and the magnitude of the splittings was found to be very similar in all of the 50 \( \mu \)m pillars studied. Figure 9(b) presents a plot of the splittings across the PL spectrum for a particular pillar. It is apparent that the spacing between the modes within the quadruplet is unequal, and decreases strongly with increasing wavelength.

The fact that the modes are always split in the 50 \( \mu \)m pillars and that the mode splitting decreases with wavelength is strong supporting evidence that the QDs are again acting as resonant scattering centres. At present, the quartet structure of the modes in unexplained. One possibility is that it arises fortuitously from a pair of doublets, with the second doublet coming from an unidentified higher-transverse-order (and lower-azimuthal-order) TE mode. To fit our data, such as that shown in Fig. 9, this other mode would, however, be required to exhibit very nearly the same free spectral range (FSR) as the first doublet. In simulations of perfectly axisymmetric pillars, we could not find any higher-transverse-order modes with FSRs that matched sufficiently closely to those of the (presumably fundamental) TE modes corresponding to the first doublet.

Inspection of both Figs 7(a) and 9(a) reveals that the background PL spectrum is modulated by a series of weak fringes, which have a periodicity that scales approximately inversely with the pillar diameter. One possible explanation for these fringes is that they originate from low-\( Q \) “triangle” type modes of the type shown schematically in Fig. 9(b). Such modes have been...
studied in deformed semiconductor laser resonators [40], and are expected to have a path length of $6R \sin(2\pi/3)$, where $R$ is the pillar radius. Given that the actual radius of the 50 $\mu$m pillar was found to be 23 $\mu$m from SEM images, we expect a path length of around 120 $\mu$m for the triangle mode, which is within 10% of the value of 110 $\mu$m determined from the spectrum.

5.3. Modelling of mode splitting

We have argued above that the scattering centres that cause the mode splitting in the larger pillars are the quantum dots themselves. In this sub-section we present the preliminary results of 2-D numerical simulations, based on the finite-element method, that support this hypothesis. The calculations were performed by COMSOL Multiphysics software, with the presence of the quantum dots encoded by a modulation in the 2-D medium’s effective refractive index. Note that the random placement of the dots breaks rotational symmetry; thus, and in contrast to previous work [41,42], the field’s (otherwise perfectly sinusoidal) azimuthal modulation cannot be separated out. Only 5 micron pillars were modelled, on account of limited computational resources, but the results that we have obtained should be equally applicable to the larger pillars.

Fig. 10. COMSOL simulations of the split modes of a 5 $\mu$m pillar containing randomly positioned QDs at a density of 50 $\mu$m$^{-2}$. The dots are represented as black circles. The mode wavelengths were (a) 974.432 and (b) 974.594 nm. The false colour depicts the electric field amplitude, with red and blue corresponding to outward and inward radial senses respectively.
The quantum dots were modelled as cylinders with a diameter of 20 nm and refractive index 3.7, as appropriate for InAs at the mode wavelength [43]. The unperturbed effective index of the mode in the micropillar was taken to be 3.34, as deduced from the fit to eqn 1. A dot density of $50 \mu m^{-2}$ was assumed, with random positioning within the WGM plane. Fig. 10 shows the results of the simulation for a split mode. The solid black circle indicates the surface of the pillar, and the false colour depicts the electric field amplitude, with red and blue corresponding to outward and inward senses respectively (the electric field being oriented predominantly radially in the plane of the pillar). The dashed cross-hairs are to aid the eye in gauging the (orthogonal) WGMs’ different orientations.

The wavelengths of the two modes shown in Fig. 10 are 974.432 and 974.594 nm. Since the QD’s permittivity is higher than that of its surrounding medium, the higher wavelength mode is the one for which a larger fraction of dots lie on the mode’s antinodal electric-field lobes (of either polarity). The calculated mode splitting of 0.16 nm is within a factor of two of that measured in the experiments. (See Fig. 6(a).) This agreement is very encouraging, given that the present model does not include the resonant nature of the QD scattering, which would be expected to increase the splitting.

We argued in Section 5.1 that the splittings observed in the 5 $\mu m$ pillar were caused by scattering by random surface roughness rather than QDs, but the simulations above indicate that the QDs should also be contributing significantly. In fact, it is probable that both types of mechanism are contributing, but that the QD scattering becomes more significant in larger pillars on account of the increased number of dots within the mode. This latter point would explain why the splitting observed for the 20 $\mu m$ pillars is significantly larger than for the 5 $\mu m$ pillars. Overall, these preliminary simulations add significant plausibility to the model that we have been proposing, although further work will be needed to obtain a close agreement between the experimental and theoretical results.

6. Conclusions

We have reported a novel experimental geometry for the observation of WGMs from GaAs/AlGaAs quantum dot micropillars with diameters ranging from 1.2 $\mu m$ to 50 $\mu m$. Multi-mode lasing behaviour is observed, with smooth thresholds characteristic of a large $\beta$ factor $> 0.5$, which is significantly higher than the value of 0.075 reported previously in similar work [20]. The gain of the laser modes is largest at the peak of the quantum dot ensemble emission. The results indicate that the WG modes have good potential as monolithic, low threshold lasers.

It is found that the WGMs observed from some of the 5$\mu m$ pillars and all of 20$\mu m$ pillars are split into doublets. In the 5$\mu m$ pillars, the observation of splitting correlates with increased surface damage and reduced $Q$, which supports their association with surface roughness [19]. The splitting of the WGMs in 20$\mu m$ pillars, by contrast, appears to be intrinsic to the structure, and varies strongly with wavelength, which is consistent with a model in which the quantum dots themselves act as resonant scattering centres. The plausibility of this model is supported by numerical simulations performed by COMSOL software. In the 50$\mu m$ diameter pillars, a more complicated mode structure is observed, but the wavelength dependence of the mode splittings is again consistent with resonant scattering by QDs. We thus conclude that WGM splitting is an inevitable feature of larger quantum dot micropillars, although further work will be needed to model the resonant nature of the scattering and to identify the mode structure of the 50 $\mu m$ pillars. We believe that this splitting mechanism should also be important in other resonant structures that incorporate quantum dots, including microdisks, rings, toroids, and microspheres.
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